FUNCTIONS

Revenue functions and Demand functions

The Revenue functions are related to Demand functions. ie. We can get the Revenue function from multiplying the demand function by quantity (x).

i.e. Revenue function = Demand function x x

Eg: 1 : If the Demand function is $2x + 3$, Calculate the Revenue function.

Revenue function = Demand function x x

= $(2x + 3) \times x$

= $2x^2 + 3x$

Eg: 2 : If the Demand function is $4x^2 + 5x - 3$, Calculate the Revenue function.

Revenue function = Demand function x x

= $(4x^2 + 5x - 3) \times x$

= $4x^3 + 5x^2 - 3x$

Total cost Functions, Variable cost functions and Fixed cost

The total cost function is included the variable cost function and fixed cost.

i.e. Total cost function = Variable cost function + Fixed cost

Note : *Here, variable cost function will be given as a form of quadric function (eg. : $1000x^2 + 8000x$) and Fixed cost will be given as a form of amount of money (eg. : Rs. 40 000/-).*

Eg: 1 : If the Variable cost function is $2x^2 + 3x$ and the fixed cost is Rs. 300,000/-, Identify the Total cost function.

Total cost function = Variable cost function + Fixed cost

= $2x^2 + 3x + 300{,}000$

Eg: 2 : If the Variable cost function is $4x^2 + 2x$ and fixed cost is Rs. 1000/-, Calculate the Total cost function.

Total cost function = Variable cost function + Fixed cost

= $4x^2 + 2x + 1000$

Profit functions

The Profit is obtained by deducting the total cost from the Revenue. Similarly the Profit function is obtained by deducting the total cost function from the Revenue function.

i.e. Profit function = Revenue function – Total cost function

Eg: 1 : If the Total cost function is $\mathbf{40 + 4x}$ and the Revenue function is $\mathbf{24x - 2x^2}$, Calculate the Profit function.

Profit function = Revenue function – Total cost function

= $24x - 2x^2 - (40 + 4x)$

= $24x - 2x^2 - 40 - 4x$

= $20x - 2x^2 - 40$

Note : *To avoid the arithmetic errors, when you substitute the values to the Total cost function it is convenient if you put bracket as in the above equation.*

Eg: 2 : The following information is given for you.

- Demand function = 120,000 + 100x
- Variable cost function = 7000x + 1000x^2
- Fixed cost = Rs. 900,000/-

By using above information, calculate the Revenue function, Total cost function and Profit function.

Revenue function = Demand function x x
= (120,000 + 100x) x x
= 120,000x + 100x^2

Total cost function = Variable cost function + Fixed cost
= 7000x + 1000x^2 + 900,000

Profit function = Revenue function – Total cost function
= 120,000x + 100x^2 – (7000x + 1000x^2 + 900,000)
= 120,000x + 100x^2 – 7000x – 1000x^2 – 900,000
= 113000x – 900x^2 – 900,000

<u>Marginal revenue functions and Marginal cost functions</u>

➢ How to calculate Marginal revenue function?

We can calculate Marginal revenue function by differentiating the Revenue function.

Eg: 1 : If the Revenue function is 5x^2 + 4x + 2000, Calculate the Marginal revenue function.

Revenue function (R) = $5x^2 + 4x + 2000$

Marginal revenue function $\frac{d(R)}{dx}$ = $(5x2)\,x^{2-1} + 4x^{1-1} + 0$
= $10x + 4 - 0$
= $10x + 4$

Eg: 2 : If the Demand function is 4x^2 + 5x – 3, Calculate the Marginal Revenue function.

To determine the marginal revenue function, we need Revenue function. But here only demand function is provided. So, first we need to find the revenue function from the demand function.

Revenue function (R) = Demand function x x
= $(4x^2 + 5x - 3)$x x
= $4x^3 + 5x^2 - 3x$

Marginal revenue function $\frac{d(R)}{dx}$ = $(4x3)x^{3-1} + (5x2)x^{2-1} - 3x^{1-1} = 12x^2 + 10x - 3$

➢ **Similarly we can calculate the Marginal cost function by differentiating the total cost function.**

Eg: 1 : If the total cost function is x^2 – 20x + 1000, Calculate the Marginal cost function.

Total cost function (TC) = $x^2 - 20x + 1000$

∴ Marginal cost function $\frac{d(TC)}{dx}$ = $2x - 20 + 0$
= $2x - 20$

Eg: 2 : If the Variable cost function is $5x^2 + 4x$ and fixed cost is Rs. 2000, Calculate the Marginal cost function.

To determine the marginal cost function, we need total cost function. But here Variable cost function and fixed cost are provided. So, first we will find the total cost function using variable cost function and fixed cost.

Total cost function (TC) = Variable cost function + Fixed cost
= $5x^2 + 4x + 2000$

∴ Marginal cost function $\frac{d(TC)}{dx} = 10x + 4 + 0 = 10x + 4$.

Break - even quantity

There are two methods to determine the Break – even quantity.

Method 1

The answers which received from the equalizing to Zero the Profit function or equalizing the Total cost function with the Revenue function is known as Break – even quantities.

Profit function = 0

or

Total cost function = Revenue function

Eg : If the Total cost function is $300x + 4800$ and the Revenue function is $-2x^2 + 500x$, Calculate the Break – even quantity.

Total cost function = Revenue function
$300x + 4800 = -2x^2 + 500x$
$2x^2 - 500x + 300x + 4800 = 0$
$x^2 - 100x + 2400 = 0$

⇒ $x = 60$ or $x = 40$
∴ Break – even quantities are 60 & 40.

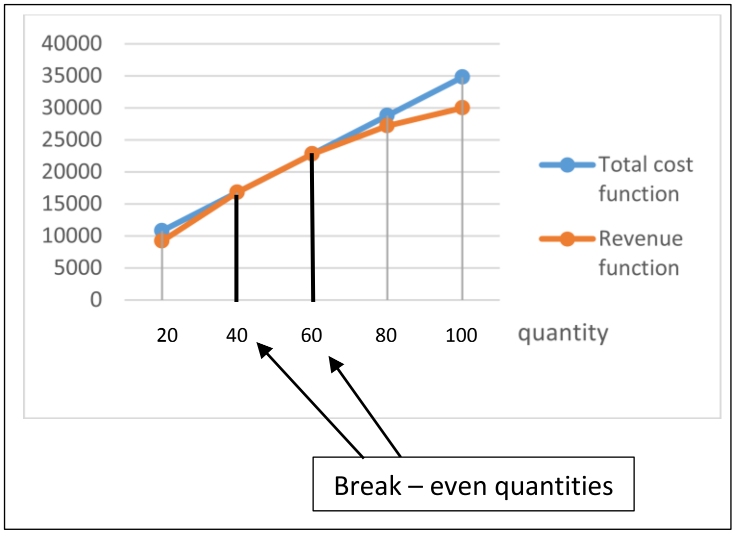
Profit function = 0
Revenue function – Total cost function = 0
$-2x^2 + 500x - (300x + 4800) = 0$
$-2x^2 + 500x - 300x - 4800 = 0$
$x^2 - 100x + 2400 = 0$
⇒ $x = 60$ or $x = 40$
∴ Break – even quantities are 60 & 40.

Method 2

Find the Break – even quantity by graphing the Total cost function and the Revenue function in the same graph.

Eg : Consider the above example. (Hint: Take the values of 20, 40, 60 80, 100 as the values of "x" for the graph)

Total cost function $300x + 4800$		**Revenue function $-2x^2 + 500x$**	
If $x = 20$ ⇒	300 x 20 + 4800 = 10,800	If $x = 20$ ⇒	$-2 \times 20^2 + 500 \times 20$ = 9,200
If $x = 40$ ⇒	**300 x 40 + 4800 = 16,800**	**If $x = 40$ ⇒**	**$-2 \times 40^2 + 500 \times 40$ = 16,800**
If $x = 60$ ⇒	**300 x 60 + 4800 = 22,800**	**If $x = 60$ ⇒**	**$-2 \times 60^2 + 500 \times 60$ = 22,800**
If $x = 80$ ⇒	300 x 80 + 4800 = 28,800	If $x = 80$ ⇒	$-2 \times 80^2 + 500 \times 80$ = 27,200
If $x = 100$ ⇒	300 x 100 + 4800 = 34,800	If $x = 100$ ⇒	$-2 \times 100^2 + 500 \times 100$ = 30,000



PROFIT MAXIMUM POINT

How to calculate the Profit maximum point?

There are two Methods of calculating the Profit Maximum point.

Method 1 (By differentiation)

At the maximum point, the ***first differentiation should be Zero*** and the ***second differentiation should be negative*** of the profit function.

Eg : If the Total cost function is $300x + 4800$ and the Revenue function is $-2x^2 + 500x$, Calculate the Profit function and the quantity at which the profit is maximized.

Profit function = Revenue function – Total cost function

$= -2x^2 + 500x - (300x + 4800)$

P $\mathbf{= -2x^2 + 200x - 4800.}$

Calculating profit maximum point :

first derivative of the profit function ***should be Zero.***

So, $\frac{d(P)}{dx} = -4x + 200 = 0 \Rightarrow 4x = 200$

$x = 50$

second derivative of the profit function ***should be negative.***

So, $\frac{d^2(P)}{dx^2} = -4 < 0$

Both conditions are satisfied.

$\therefore$ $x = 50$.

Method 2 (By using break – even points)

Eg 1 : If the Total cost function is $300x + 4800$ and the Revenue function is $-2x^2 + 500x$, Calculate the Profit function and the quantity at which the profit is maximized.

At the Break – even point.

Total cost function = Revenue function

$300x + 4800 = -2x^2 + 500x$

$2x^2 - 500x + 300x + 4800 = 0$

$x^2 - 100x + 2400 = 0 \Rightarrow x = 60$ or $x = 40$

$\therefore$ Break – even quantities are 60 & 40.

So, the quantity at which Profit is maximized $= \frac{60+40}{2} = 50$ $\therefore x = 50.$

Eg 2 : Weekly **profit function** of a company is given by $\mathbf{P = 1{,}400x - x^2 - 240{,}000}$ where x is the number of units produced per week. How many units to be sold to maximize the weekly profit?

$P = -x^2 + 1400x - 240{,}000 = 0$

$\frac{d(P)}{dx} = -2x + 1400 = 0 \Rightarrow 2x = 1400 \Rightarrow x = 700$

$\frac{d^2(P)}{dx^2} = -2 < 0$ $\therefore x = 700.$

EXERCISES

01. The variable cost of a manufacturing company is Rs.6/- per unit and the total fixed cost is Rs.560/-. The total revenue function is given below:

$TR = -2x^2 + 30x + 520$ where x is the number of units produced.

(i) Find the Profit Function.

(ii) Calculate the quantity at which the profit is maximized using differentiation.

Answer:

(i) Profit function = Revenue function – Total cost function

= Revenue function – (Variable cost function + Fixed cost)

$= -2x^2 + 30x + 520 - (6x + 560)$

$= -2x^2 + 30x + 520 - 6x - 560$

$P = \mathbf{-2x^2 + 24x - 40}$

(ii) Profit function (P) $= -2x^2 + 24x - 40$

$\frac{d(P)}{dx} = -4x + 24 = 0 \Rightarrow 4x = 24 \Rightarrow x = 6$

$\frac{d^2(P)}{dx^2} = -4 < 0$ $\therefore x = 6$

02. One of the machineries of a company is capable of producing a maximum of 10,000 units per week. The weekly cost to produce "x" No. of units is given by,

$TC = 75{,}000 + 100x - 0.03x^2 + 0.000004x^3$

and the demand function for the units is $D = 200 - 0.005x$

Identify the marginal cost, marginal revenue and marginal profit functions.

Answer:

(i) Total cost function (TC) $= 75{,}000 + 100x - 0.03x^2 + 0.000004x^3$

Marginal cost function $= \frac{d(TC)}{dx} = 0 + 100 - 0.06x + 0.000012x^2$

$= 100 - 0.06x + 0.000012x^2$

(ii) Revenue function (R) = Demand function x x

$= (200 - 0.005x) \times x = 200x - 0.005x^2$

Marginal Revenue function $= \frac{d(R)}{dx} = 200 - 0.01x$

(iii) Profit function (P) = Revenue function – Total cost function

$= 200x - 0.005x^2 - (75{,}000 + 100x - 0.03x^2 + 0.000004x^3)$

$= 200x - 0.005x^2 - 75{,}000 - 100x + 0.03x^2 - 0.000004x^3$

$= 100x + 0.25x^2 - 75{,}000 - 0.000004x^3$

Marginal Profit function $= \frac{d(P)}{dx} = 100 + (0.25 \times 2)x^{2-1} - 0 - (0.000004 \times 3)x^{3-1}$

$= 100 + 0.5x - 0 - 0.000012x^2$

$= 100 + 0.5x - 0.000012x^2$

03. Cost function and revenue function of a company are as follows, where x is the number of units produced and sold:

$TR = 8x$, $TC = 6x + 1{,}400$

Calculate the break-even number of units.

Answer:

(i) At the Break – even point,

Total cost function = Revenue function

$6x + 1{,}400 = 8x$

$8x - 6x = 1400$

$2x = 1400 \Rightarrow x = 700$

04. Calculate the following using the given data below.

(i) Revenue function
(ii) Total cost function
(iii) Profit function
(iv) Marginal Revenue function
(v) Marginal cost function
(vi) Break – even point

Demand function = $3x + 7$

Variable cost function = $3x^2 - 3x$

Fixed cost = Rs. 60/-

Answer:

(i) Revenue function = Demand function x x

$= (3x + 7) \times x$

$= 3x^2 + 7x$

(ii) Total cost function = Variable cost function + Fixed cost

$= 3x^2 - 3x + 60$

(ii) Profit function = Revenue function – Total cost function

$= 3x^2 + 7x - (3x^2 - 3x + 60)$

$= 3x^2 + 7x - 3x^2 + 3x - 60$

$= 10x - 60$

(iv) Marginal Revenue function $= \frac{d(R)}{dx} = 6x + 7$

(v) Marginal cost function $= \frac{d(TC)}{dx} = 6x - 3$

(vi) Break even point ⇒ Revenue function = Total cost function

$3x^2 + 7x = 3x^2 - 3x + 60$

$10x = 60$

$\Rightarrow x = 6$